THE IMPACT OF THE UNAVAILABILITY OF LINE REPLACEABLE UNITS ON THE OPERATIONAL AVAILABILITY OF SYSTEM

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Abstract

The evaluation of the operational availability of a fleet of systems on an operational site is far from trivial when the size of the state space of a faithful Markovian model makes this issue unrealistic for many large fleet of systems. The main difficulty comes from the existence on the site of line replaceable units that may be unavailable from time to time when a breakdown occurs. In this paper, we present a simpler but approximate method which has given a quite good accuracy when tested on small cases. The main idea is to consider a non product form queuing network and to aggregate subsets of it as if they were parts of a product form queuing network. Nevertheless, the generalization to systems with multiple types of line replaceable units needs to be investigated further and tested with respect to the accuracy of the new approximate method.

Keywords: integrated logistic system, stock shortage, line replaceable unit, operational availability, intrinsic availability, Markovian model, approximate method, non product form queuing network.

1 INTRODUCTION

Engineers who design new equipment do everything in their power to make this equipment reliable. This means they aim to achieve a low failure rate; possibly at the cost of very good quality components and / or by the establishment of redundant structures in the sense of reliability. The limits of this approach being, on the one hand the level of their expertise and on the other hand, the cost of the future equipment. In this study, we focus on the performance of (often complex) systems used in multiple copies on the same operational site and therefore on the prediction of the operational availability of the fleet assigned on the site. As far as possible during the design phase, the reliability engineers have estimated an intrinsic availability.

The intrinsic availability is an upper limit of the operational availability because the considered unavailability corresponds only to the time necessary to proceed with the exchange of the defective element. This assumes that the repairer and the spare part are always immediately available on the operational site. To reduce intervention time at an operational site, system repair consists of replacing the defective subset with an identical one in good condition. These exchangeable subsets are called LRUs (for ”line replaceable units”). Restoring a piece of system by exchanging an LRU makes it possible to obtain a rapid return to service of the system and does not require the presence on the operational site of several specialists expert in their own field of expertise. Thus, we can change an aircraft engine in a few hours thanks to stakeholders who are knowledgeable about the procedures for intervention on fasteners and connections but who are not required to know the techniques to repair a broken engine. The operational availability (the one that is of interest to the user) will generally be lower than the intrinsic availability because the latter integrates the unavailability of repairer or LRU, if any. And if the waiting time for a repairer is generally measured in hours, the unavailability of a LRU can be measured in days, in weeks, even in months! It is therefore this last point which a potential customer should worry about in priority. Moreover, the unavailability of the LRU is, as we shall see, more difficult to model and evaluate than that of the repairer.

To try to predict the operational availability of a future fleet of systems, several directions of research can be considered. Thus, a simulation of the logistic support system, represented by a discrete event system, can be considered; but the number of event types to consider and the total number of events to simulate (to obtain reliable results) will generally be ground for abandoning this direction. A second possible direction is that of a Markovian model of the support system. But if this path is attractive to deal with the case of a subsystem, there is an explosion in the number of possible states when trying to deal with the case of many support systems. There remain the directions of the so-called approximate methods which can be very acceptable when the parameters remain confined to a given subspace of the physically admissible space of the said parameters. Unfortunately, the acceptable subspace seems very difficult to characterize! This is why in this study we consider a virtual system with only one type of LRU in order to understand the influence of various parameters on the operational availability both of a faithful Markovian model (for which we are able to get the exact answer) and of a proposed approximate method. By doing so, we are able to compute the relative errors induced when using this latter solution. Note that if the spare LRUs were always available on the operational site (which is never the case) we could fairly model the behavior of the support system on the site by means of a so called product form queueing network, a well known class of queueing networks.

The paper is organized as follow. In a first step (Section 2), we build a Markovian model to take into account the presence of a limited stock of LRUs on the site. The model is relatively simple since, as we said, we consider a unique type of LRU. Then we use this model to better understand the variation of the operational availability according to the different parameters (Section 3). In Section 4, we first introduce a new approximate method for calculating the operational availability and then we test the influence of the parameters on the relative error thanks to the Markov model. Finally, we conclude this paper by summerizing the obtained results and by mentionning potential extensions.

2 DETERMINATION OF A MARKOVIAN MODEL

As mentionned in the introduction, we only consider the case of a single type of LRU equipment in order to have a probabilistic model that is as simple as possible while behaving in a way that is almost similar to a real system. Note that the LRU equipment is the only subset of the virtual system that can break down. Suppose the fleet consists of M (very small) systems. Let n_1, n_2 and n_3 be the number of systems that are, at a given time, 1) in operating state, 2) unavailable and waiting for a new LRU and 3) unavailable and in the process of exchanging their LRU, respectively. We assume that the operating time and the exchange time of an LRU follow exponential probability distributions of respective λ and μ rates. Let's define s as the number of repairers assigned to support this fleet. To reduce the waiting time for a LRU in good condition, a (small) stock of R LRUs in good condition is initially assigned to to the operational site.

The modeling and the evaluation of the expectation of a potential waiting time for a spare LRU is less straightforward. To introduce the difficulty, let us begin by presenting an acceptable Markovian model which would give good results if the exchange rate μ was infinite (this assumes that n_3 is always null). In such a situation, the Markovian model is a Birth and Death process (BDP) whose possible states are the integers $0, 1, 2, ..K$, where $K = M + R$. Here, breakdowns of systems are associated to death events of the BDP and the order deliveries are associated to birth events of the BDP. Given that the BDP is in state k, then n_1, the number of systems in operating state equals $(M - (M - k)^+)$, where $(M - k)^+ = \max(0, (M - k))$. The transition rate of a death event equals $n_1\lambda$, *i.e.*, $(M - (M - k)^+)\lambda$. Consequently, n_2, the number of systems wanting for a LRU in good condition, equals $(M - n_1) = (M - k)^+$ when this BDP is in state k. Also the number of LRUs awaiting delivery equals $(K - k)$. Here we assume that the transition rate $\beta(k)$ of a birth event from state k to state $(k + 1)$ depends only on $(K - k)$, the number of LRUs awaiting delivery.

The determination of transition rate $\beta(k)$ of a birth event is not straightforward and is developed thereafter. First, let us imagine a process running through a source-terminal diagram describing the timed steps that are produced following the issuance of the order of an LRU, until delivery to the operational site of the LRU. This process has a random duration which corresponds to the travel duration on an oriented random graph. Now imagine that delivery of the LRU immediately triggers a new LRU order. We would then have a loop traversed by an order whose rate of passage on the operational site would be the opposite of the expected duration of travel of the loop. Let's define $Beta(1)$ this rate of transition. Suppose now that k orders circulate in this loop permanently. The rate of passage on the operational site of these orders, noted $Beta(k)$, will be greater than $Beta(1)$. We assume that $Beta(k + 1) \geq Beta(k)$ whatever is k and that $Beta(k) \leq kBeta(1)$. The case $Beta(k) = kBeta(1)$ corresponds to an organization whose shared resources are sufficient to avoid waiting time (for example the faulty LRU is not blocked due to the unavailability of a repairer).

Let's suppose that thanks to a rough model solved by pen and paper or by simulation, we can estimate the values of the $Beta(k)$, $k = 1, 2, ..., K$; then the procedure is to take $\beta(k) = Beta(K - k)$, $k = 0, 1, 2, ..., (K - 1)$. In this paper, in order to facilitate the interpretation of the results, we examine the specific situation where

$\beta(k) = (K-k)Beta(1)$. This means that $Beta(1)$ will be a parameter that we will denote β. The Markovian graph of this model is represented in Figure 1.

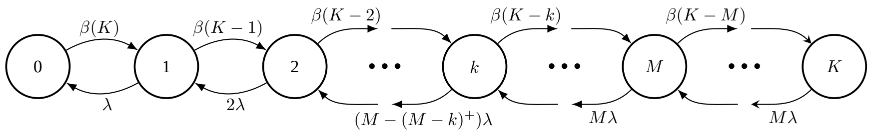

Figure 1: Transition graph of the BDP.

Now that we have worked out the model relative to the LRU delivery mechanism, we consider the overall model which includes the variable n_3 (when μ is finite). This overall model is a continuous time Markov chain (CTMC) such that a state of this CTMC corresponds to a couple (n_1, k) belonging to the set $\{(n_1, k) \,|\, 0 \le n_1 \le M, n_1 \le k \le K\}$. Note that if n_1 and k are known, then $n_3 = M - n_1 - (M-k)^+$. This is the number of unavailable systems for which the repairers are in the process of exchanging their LRU.

If $a_{e;e'}$ denotes an element of the infinitesimal generator of the CTMC, then the non-null non-diagonal elements are given by the following relations:

$$a_{(n_1,k);(n_1-1,k)} = n_1\lambda \qquad \text{if} \quad k > M \tag{1}$$

$$a_{(n_1,k);(n_1-1,k-1)} = n_1\lambda \qquad \text{if} \quad k \le M$$

$$a_{(n_1,k);(n_1,k-1)} = \beta(k) \qquad \text{if} \quad k < K$$

$$a_{(n_1,k);(n_1+1,k)} = \mu(n_3) \quad \text{if} \quad n_3 > 0\ ,$$

where $\mu(n_3) = \inf(s, n_3)\mu$ and $n_3 = M - n_1 - (M-k)^+$.

The knowledge of this infinitesimal generator allows us to compute the steady-state probability distribution of the CTMC [2, 3, 4] and from that to determine the steady-state availability. This solution will serve as the reference point when we will be testing the approximate method that will be introduced in Section 4.

In Section 3 below, we use this first model to examine its sensitivity with respect to different parameters.

3 SENSITIVITY WITH RESPECT TO THE DIFFERENT PARAMETERS

If not mentioned otherwise, we will consider the normalized availability, *i.e.*, the mean number of available systems divided by M, the total number of systems corresponding to the fleet.

We first look at the sensitivity of the Markovian model with respect to M when the total failure rate $M\lambda$ stays constant.

In Figure 2 we investigate the influence of M on the availability when the product $M\lambda$ stays constant and M increases from $M = 1$ to $M = 50$. The product $M\lambda$ stays equal to 0.06. Parameters β and μ are respectively equal to 0.05 and 0.2. The availability is plotted for $R = 0, 1, 2$. As expected, the availability increases with the value of R. When $M = 1$ and $R = 0$, the mean operational time equals $1/\lambda = 1/0.06$ while the mean non-operational time equals $(1/\beta + 1/\mu) = (1/0.05 + 1/0.2)$. The corresponding value of the availability is 0.4. When $M = 50$ and $R = 0$, the mean operational time equals $1/\lambda = 1/0.012$ while the parameters β and μ keep the same values. Since the total failure rate does not increase, the unique repairer keeps a small utilization ratio and the mean rate of orders of LRU does not increase either. As a consequence, the mean non-operational time does not increase significantly while the mean operational time is multiplied by 50. This involves a significant increase of the availability.

In Figure 3 we investigate the influence of M on the percentage of unavailability due to the lack of good LRUs again when the product $M\lambda$ stays constant and M increases from $M = 1$ to $M = 50$. We plotted the ratio of the unavailability due to the lack of good LRUs over the total unavailability. When $M = 1$ and $R = 0$, this ratio corresponds to $(1/\beta)/(1/\beta + 1/\mu)$ which is here equal to 0.8. We can see in this figure that the ratio decreases when M increases. This is due to the fact that when $R = 0$, the percentage corresponding to the delivery of good LRUs behaves as a queue with an infinite number of servers while the limited number of

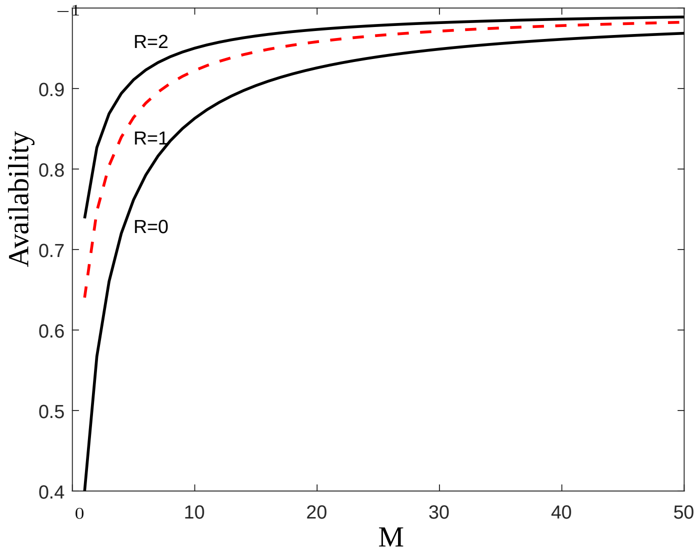

Figure 2: Availability for $R = 0, 1, 2$. Influence of M when the product $M\lambda$ stays equal to 0.06.

assigned repairers induces some extra waiting time in the exchange process. When R takes a non-null value, the behaviour of the delivery of good LRUs does not behave as a classical queue and the evolution of the ratio w.r.t. M has no obvious explanation.

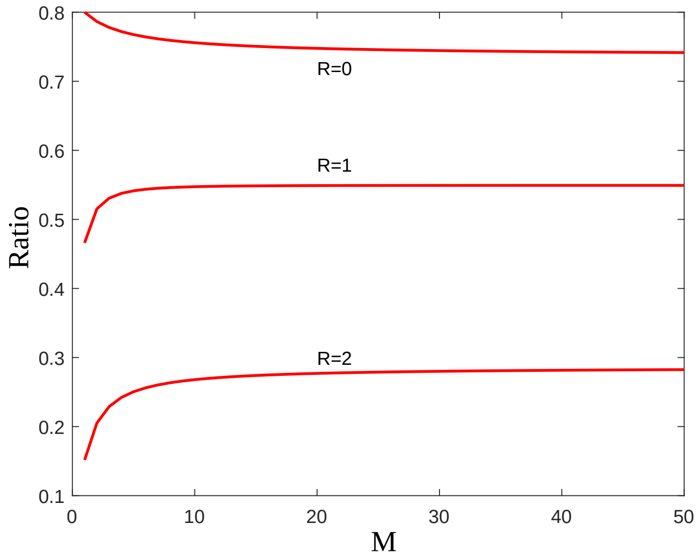

Figure 3: Unavailability ratio for $R = 0, 1, 2$. Influence of M when the product $M\lambda$ stays equal to 0.06.

In Figure 4 we consider the variation of parameter β from 0.006 to 0.8. We plotted the availability for $R = 0, 1, 2$ and 3. In this figure, the parameters with fixed values are $M = 30$, $\lambda = 0.002$, $\mu = 0.02$ and $s = 4$. We remark that when β is large enough, the availability tends to a maximal value of 0.893 that corresponds to the limit when β tends to infinity. In that situation, the remaining unavailability is the consequence of the time to exchange the LRU. Note that when $R = 3$, this maximal value is practically reached when $\beta = 0.02$. In other words, assuming $\beta = 0.02$ and $R = 3$, then the lack of LRUs has a negligible influence on tne unavailability w.r.t. that of the exchanging time.

If we had plotted the variation of availability w.r.t. parameter μ, we would have observed a higher limit of the availability for $R = 3$ and $\beta \geq 0.02$. For example, the availability equals 0.98 when $\mu = 0.1$ and 0.998 when $\mu = 1$.

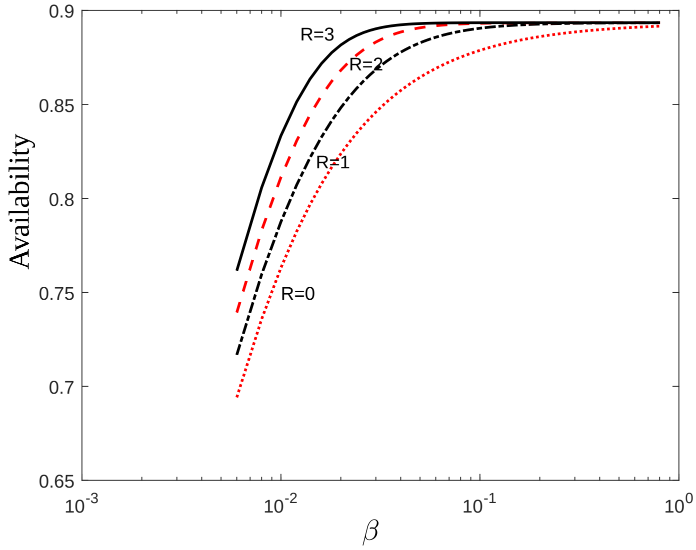

Figure 4: Availability for $R = 0, 1, 2$ and 3. Influence of β.

4 A NEW APPROXIMATE METHOD

4.1 The model

We consider a queuing network with three stations. Station 1 models the operational systems, with an individual failure rate λ. Station 3 models the exchanging operation of the failed LRU where each repairer is a server with rate μ. These two stations are of the class of product form stations. Station 2 is the one that makes the queuing network not a product form one (except when $R = 0$).

The idea is to aggregate the sub-network composed of the two product form stations in order to create a single station as if these two stations were part of a product form queuing network. It is shown in [1] that this aggregation would not modify the initial solution of the complementary network if the queuing network was a product form one. Following [1], the service rate $\nu_a(i)$ of the aggregated station can be expressed as:

$$\nu_a(i) = c_a \frac{G(i-1)}{G(i)} \qquad i = 1, 2, ..., M\,, \tag{2}$$

where, in the special case of stations 1 and 3,

$$G(i) = \sum_{j=0}^{i} \frac{X_3^j}{A_3(j)} \frac{X_1^{i-j}}{(i-j)\,!} \qquad i = 1, 2, ..., M \tag{3}$$

$$X_1 = c_a/\lambda$$

$$X_3 = c_a/\mu\,,$$

c_a is an arbitrary constant and

$$A_3(j) = \begin{cases} \frac{1}{j!} & \text{if } j \leq s \\ \frac{1}{s\,!s^{j-s}} & \text{if } j \geq s \end{cases}\,, \tag{4}$$

In addition, note that $A_3(0) = 1$ and $G(0) = 1$.

Once the sequence $\nu_a(i)$ has been computed, the last phase of the proposed method consists in studying the special station 2 that we introduced in Section 2 with a new arrival rate of failed LRUs:

$$\lambda(k) = \nu_a(M - (M-k)^+) \qquad k = 1, 2, ..., K\,, \tag{5}$$

In the following section, we test the accuracy of this approximate method using the solution given in Section 2.

4.2 Experimental results

Since this approach gives the exact availability when R equals to zero, we only consider positive values of R while performing this experiment.

In Figure 5 we examine the evolution of the relative error introduced by the proposed method on the availability when the parameter β varies from 0.006 to 0.8. We consider the cases $R = 1, 2$ and 3. We observe that the three curves are concave and that they realize their maximal values for distinct values of β. The higher maximal value is close to 10^{-3} and occurs when $R = 3$. That would allow us to estimate very accuratly the operational availability of a real fleet of systems. In addition, in the real world on an operational site, the values of R are generally either $R = 0$ or $R = 1$ and rarely greater than one. That moderates the fact that the maximal value seems to be increasing w.r.t. parameter R. In this figure, the parameters with fixed values are $M = 30$, $\lambda = 0.002$, $\mu = 0.02$ and $s = 4$.

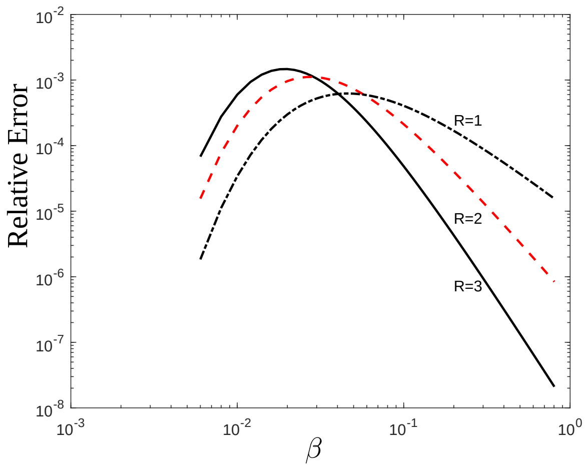

Figure 5: Variation of the relative error on the availability w.r.t. parameter β (absolute value).

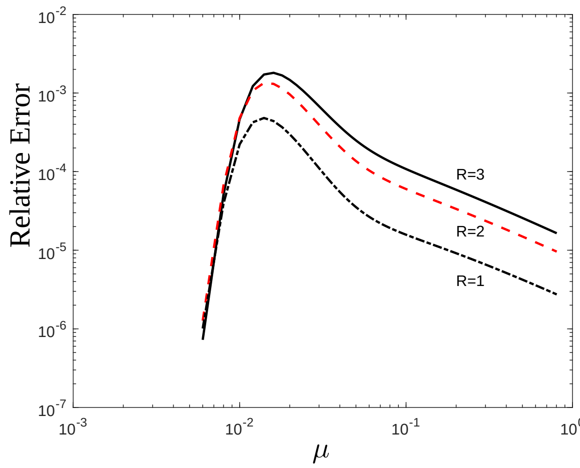

Figure 6: Variation of the relative error on the availability w.r.t. parameter μ (absolute value).

In Figure 6 we examine the evolution of the relative error introduced by the proposed method on the availability when the parameter μ also varies from 0.006 to 0.8. Let's consider once more the cases where $R = 1, 2$ and 3. We observe that the three curves are no longer concave but still uni-modal and that they realize their maximal values for values of μ which are very close to each other. The higher maximal value is also obtained when $R = 3$ and is close to 10^{-3}. In this figure, the parameters with fixed values are $M = 30$, $\lambda = 0.002$, $\beta = 0.02$ and $s = 4$.

5 CONCLUSION

Due to the explosion of the state-spaces of classical Markovian models when evaluating operational availability of fleet of systems of real size, with a risk of shortage of LRUs, we presented a new approximate method. So far, this approach has been tested on a simple system reduced to a single type of LRU. We used a classical Markovian model (a continuous time Markov chain) to serve as a reference point to test the new approximate method. Furthermore the Markovian model furnished numerical results that contributed to understand the behavior of the logistic of the fleet of systems. This new approach gives promising results by providing low relative errors on the approximated operational availability. Hence the generalization to systems with multiple types of LRU needs to be investigated further and tested with respect to the accuracy of the new approximate method.

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